

Cosmological Constant of the $(p + 1)$ -Dimensional World, Embedded in the d -Dimensional Bulk Space

Davoud Kamani

Received: 11 August 2007 / Accepted: 11 October 2007 / Published online: 23 October 2007
© Springer Science+Business Media, LLC 2007

Abstract In this manuscript we study the cosmological constant of a $(p + 1)$ -dimensional world, which lives in the higher dimensional bulk space. We assume the extra dimensions are compact on tori. We consider two cases: positive and negative bulk cosmological constant. It is pointed out that the tiny cosmological constant of our world can be obtained by the dynamics of a scalar field and adjusting the parameters of the model. The cosmological constant of the dual world also will be discussed. We obtain the Dirac quantization of these cosmological constants.

Keywords Cosmological constant · Compactification · Brane

1 Introduction

From the 4-dimensional point of view it seems that a solution for the problem of the cosmological constant is practically impossible [1]. Therefore, various attempts have been made to explain the mystery of the cosmological constant, but no satisfactory solution has been found yet. In other words, “in the standard framework of low energy physics there appears to be no natural explanation for vanishing or extreme smallness of the vacuum energy”, as Witten put it in [2]. Braneworlds (e.g. see [3–8]) offer new tools that could potentially solve the cosmological constant problem. The appearance of extra dimensions in the framework of braneworld systems seems to provide some new ideas to address this problem from a different point of view (e.g. see [3–5, 9–11] and references therein). For reviewing the various problems of the cosmological constant see [1, 12–17].

From the other side, a rolling scalar with a nearly flat potential provides another useful tool for studying the cosmological constant [18–22]. The potential must be very flat with some effective continuous variation in order that the vacuum energy be constant.

In this manuscript we consider a Dp -brane in the d -dimensional bulk space, with both positive and negative bulk cosmological constant, as the general set-up. Generally, this

D. Kamani (✉)

Faculty of Physics, Amirkabir University of Technology, P.O. Box: 15875-4413, Tehran, Iran
e-mail: kamani@ic.aut.ac.ir

model is not like the standard braneworld models. Particular emphasis will be put on the compactification of the $d - p - 1$ extra dimensions. In our warped model the cosmological constant of the $(p + 1)$ -dimensional universe is not an arbitrary constant. It is calculable through the moduli of the model. That is, any physical mechanism which defines (stabilizes) the bulk cosmological constant, brane tension and radii of compactification together with the dynamics of the metric of the extra space will simultaneously determine the cosmological constant.

We observe that a scalar field, generated by the metric of the extra part of the bulk space, generally controls the other parameters of the theory. That is, when the bulk cosmological constant and the brane tension go to zero and the radius of compactification goes to infinity when the scalar field goes to infinity the mechanism of vanishing of the cosmological constant generally turns off.

For matching with the real world, the D3-brane of the type IIB superstring theory will be emphasized. Therefore, the tiny cosmological constant of the 4-dimensional world is a consequence of the dynamics of the scalar field and adjusting of the parameters of the theory.

Finally, cosmological constant of the $(d - p - 3)$ -dimensional dual world will be studied. The cosmological constants of the world and its dual world obey the Dirac quantization.

This paper is organized as follows. In Sect. 2, the cosmological constant of the model will be obtained. In Sect. 3, the cosmological constant of the model with positive bulk cosmological constant will be analyzed. In Sect. 4, the cosmological constant due to the negative bulk cosmological constant will be studied. In Sect. 5, the cosmological constant of the dual world will be discussed. Section 6 is devoted for the conclusions.

2 The Cosmological Constant of the Model

Consider a Dp -brane in the d -dimensional spacetime (bulk space). The corresponding action is given by

$$S = S_g + S_{\text{DBI}}, \tag{1}$$

where the first term is the d -dimensional gravitational action and the second term is the Dirac-Born-Infeld (DBI) action corresponding to the Dp -brane. Since the Chern-Simons action does not contribute to our calculations, we ignore it.

The gravitational action, with the bulk cosmological constant Λ_d , is

$$S_g = \frac{1}{2\kappa_d^2} \int d^d x \sqrt{-\det G_{\mu\nu}} (\mathcal{R}_d - 2\Lambda_d), \tag{2}$$

where $\mu, \nu \in \{0, 1, \dots, d - 1\}$. The d -dimensional gravitational constant κ_d in terms of the Newton's constant G_d is $\kappa_d^2 = 8\pi G_d$. In this action, up to the cosmological constant Λ_d , we assumed that the graviton is the only bulk field.

For the Dp -brane the DBI action is

$$S_{\text{DBI}} = -T_p \int d^{p+1} \xi \sqrt{-\det(g_{\alpha\beta} + 2\pi\alpha' F_{\alpha\beta})}, \tag{3}$$

where $\alpha, \beta \in \{0, 1, \dots, p\}$. Here $g_{\alpha\beta}$ is the induced metric on the brane and $F_{\alpha\beta}$ is the strength of the $U(1)$ gauge field living on the brane. Since the brane lives in the

d -dimensional spacetime, its tension has the form

$$T_p = \frac{\sqrt{\pi}}{2^{(d-10)/4} \kappa_d} (4\pi^2 \alpha')^{(d-2p-4)/4}. \tag{4}$$

Expansion of the DBI action gives the following terms

$$S'_{\text{DBI}} = -T_p \int d^{p+1} \xi \sqrt{-\det g_{\alpha\beta}} \left(1 + \frac{1}{4} (2\pi \alpha')^2 F_{\alpha\beta} F^{\alpha\beta} + \dots \right). \tag{5}$$

The second term is the Yang-Mills action in the curved background. Therefore, its coefficient defines the Yang-Mills coupling. The first term also will be used in calculating the cosmological constant.

2.1 Compactification of the Extra Dimensions of the Bulk Space

In our model the worldvolume of the Dp -brane describes a $(p + 1)$ -dimensional universe. Thus, the bulk space has the $d - p - 1$ extra dimensions. Assume that all these directions are compact on a tori with equal radii R . In addition, let the fields be independent of the extra coordinates. Furthermore, we assume that all the brane directions are non-compact. Therefore, by the standard Kaluza-Klein reduction the action (2) reduces to

$$S'_g = \frac{(2\pi R)^{d-p-1}}{2\kappa_d^2} \int d^{p+1} x \sqrt{-\det G_{\alpha\beta}} e^{-2\phi_{p+1}} \times \left(\mathcal{R}_{p+1} - 2\Lambda_d + 4\partial_\alpha \phi_{p+1} \partial^\alpha \phi_{p+1} - \frac{1}{4} G^{mn} G^{pq} \partial_\alpha G_{mp} \partial^\alpha G_{nq} - \frac{1}{4} G_{mn} F_{\alpha\beta}^m F^{n\alpha\beta} \right), \tag{6}$$

where the indices m and n run over the compact coordinates, the scalar field is $\phi_{p+1}(x^\alpha) = -\frac{1}{4} \ln \det G_{mn}$ and $F_{\alpha\beta}^m$ is field strength of the gauge bosons $G_{m\alpha}$. As we see, the scalar field comes from the compactness of the extra dimensions. In addition, ϕ_{p+1} and G_{mn} are not independent fields. We shall see that this scalar field plays an important role in the cosmological constant of the $(p + 1)$ -dimensional world.

Let the brane be along the non-compact coordinates of the bulk space. Thus, the embedding functions are $x^\alpha(\xi) = \xi^\alpha$, and hence $g_{\alpha\beta} = G_{\alpha\beta}$. Besides, we make the following field redefinition

$$\tilde{G}_{\alpha\beta} = \exp \left(-\frac{4}{p-1} \phi_{p+1} \right) G_{\alpha\beta}, \tag{7}$$

which is the Weyl transformation. The metric $\tilde{G}_{\alpha\beta}$ is called the Einstein metric. Introducing the metric (7) into (5) and (6) and summing the resulted actions lead to

$$S' = \int d^{p+1} \xi \sqrt{-\det \tilde{G}_{\alpha\beta}} \left[\frac{(2\pi R)^{d-p-1}}{2\kappa_d^2} \tilde{\mathcal{R}}_{p+1} - \frac{(2\pi R)^{d-p-1}}{\kappa_d^2} \Lambda_d \exp \left(\frac{4}{p-1} \phi_{p+1} \right) - T_p \exp \left(\frac{2(p+1)}{p-1} \phi_{p+1} \right) + \frac{(2\pi R)^{d-p-1}}{2\kappa_d^2} \exp \left(\frac{4}{p-1} \phi_{p+1} \right) \times \left(4\partial_\alpha \phi_{p+1} \partial^\alpha \phi_{p+1} - \frac{1}{4} G^{mn} G^{pq} \partial_\alpha G_{mp} \partial^\alpha G_{nq} - \frac{1}{4} G_{mn} F_{\alpha\beta}^m F^{n\alpha\beta} \right) + \dots \right], \tag{8}$$

where $\tilde{\mathcal{R}}_{p+1}$ is the scalar curvature corresponding to the metric $\tilde{G}_{\alpha\beta}$.

The first three terms of this action have the feature of the action (2). Thus, it enables us to read the constants. The first term defines the gravitational coupling of the $(p + 1)$ -dimensional world

$$\kappa_{p+1}^2 = (2\pi R)^{p+1-d} \kappa_d^2. \tag{9}$$

Since $\kappa^2 = 8\pi G_{\text{Newton}}$, the Newton’s coupling constants find the relation

$$G_{p+1} = (2\pi R)^{p+1-d} G_d. \tag{10}$$

In the same way, the second and the third terms give the cosmological constant of the $(p + 1)$ -dimensional world, i.e. $\Lambda_{p+1}^{(d)}$,

$$\begin{aligned} \Lambda_{p+1}^{(d)} &= \Lambda_{\text{bulk}}^{(d)} + \bar{\Lambda}_{p+1}^{(d)}, \\ \Lambda_{\text{bulk}}^{(d)} &= \Lambda_d \exp\left(\frac{4}{p-1} \phi_{p+1}\right), \\ \bar{\Lambda}_{p+1}^{(d)} &= 8\pi G_{p+1} T_p \exp\left(\frac{2(p+1)}{p-1} \phi_{p+1}\right). \end{aligned} \tag{11}$$

The term $\Lambda_{\text{bulk}}^{(d)}$ is the cosmological constant due to the bulk space, and $\bar{\Lambda}_{p+1}^{(d)}$ originates from the Dp -brane. Since ϕ_{p+1} is a dynamical field, the phrase “cosmological potential” for $\Lambda_{p+1}^{(d)}$ is more suitable, but we use the common phrase “cosmological constant”.

The compactification influences both components of the cosmological constant. The effects of the compact part of the bulk space are given by ϕ_{p+1} , and the radius R through (10).

The cosmological constant $\Lambda_{p+1}^{(d)}$ depends on the parameters Λ_d , R and T_p . Thus, by adjusting these parameters, the shape of the potential $\Lambda_{p+1}^{(d)}(\phi_{p+1})$ will be fixed. Then, the dynamical property of ϕ_{p+1} enables us to receive the desired value of $\Lambda_{p+1}^{(d)}$.

The vacuum energy density of the $(p + 1)$ -dimensional world is as in the following

$$\begin{aligned} \rho_{p+1} &= \rho_d (2\pi R)^{d-p-1} \exp\left(\frac{4}{p-1} \phi_{p+1}\right) + T_p \exp\left(\frac{2(p+1)}{p-1} \phi_{p+1}\right), \\ \rho_{p+1} &\equiv \frac{\Lambda_{p+1}^{(d)}}{8\pi G_{p+1}}, \quad \rho_d \equiv \frac{\Lambda_d}{8\pi G_d}, \end{aligned} \tag{12}$$

where ρ_d is the vacuum energy density of the bulk space. We observe that the vacuum energy of the world is not sum of the vacuum energies of the brane and bulk. In addition, finiteness of the vacuum energy density of the world implies that the extra dimensions have to be compact. In other words, the radius R cannot go to infinity.

3 Cosmological Constant Due to $\Lambda_d > 0$

We observe that for the positive Λ_d , the world cosmological constant $\Lambda_{p+1}^{(d)}$ always is a positive quantity. In other words, positivity of Λ_d gives a continues form to $\Lambda_{p+1}^{(d)}$, from zero to infinity, which are corresponding to $\phi_{p+1} = -\infty$ and $\phi_{p+1} = +\infty$, respectively.

For sufficiently small Λ_d and large extra dimensions, when ϕ_{p+1} has its small values, the cosmological constant is small and positive. The other interesting case is as follows. For any values of the parameters, when $\phi_{p+1} \rightarrow -\infty$ the cosmological constant, from the positive side, goes to zero.

Now consider $\Lambda_d \rightarrow 0$. Therefore, when the radius of compactification goes to infinity and the scalar field is at infinity the mechanism of vanishing of the cosmological constant generally turns off.

3.1 Four-Dimensional Universe

The Dp -branes of the superstring theory, i.e. $d = 10$, give

$$\Lambda_{p+1}^{(10)} = \Lambda_{10} \exp\left(\frac{4}{p-1}\phi_{p+1}\right) + \frac{g_s}{8\pi^2\alpha'} \left(\frac{\sqrt{\alpha'}}{R}\right)^{9-p} \exp\left(\frac{2(p+1)}{p-1}\phi_{p+1}\right), \tag{13}$$

where $\kappa_{10}^2 = 8\pi G_{10} = \frac{1}{2}(2\pi)^7 g_s^2 \alpha'^4$ has been used.

As an interesting example, for the D3-brane of the type IIB superstring theory the corresponding cosmological constant is

$$\Lambda_4^{(10)} = \Lambda_{10} e^{2\phi_4} + \frac{g_s \alpha'^2}{8\pi^2 R^6} e^{4\phi_4}. \tag{14}$$

For an appropriate value of ϕ_4 , i.e. $\phi_4 = \phi_0$, this should match with the observed cosmological constant of our 4-dimensional world. That is, $\Lambda_4^{(10)}(\phi_0)$ should be positive and small. This is done by adjusting the moduli Λ_{10} , g_s , R and ϕ_0 . In other words, the dynamical property of ϕ_4 provides a dynamical mechanism for adjusting the cosmological constant to an almost zero value. Therefore, in the present time, the universe is in the one of the following regime: (1) $\phi_4 \rightarrow \phi_0 = -\infty$ and the parameters Λ_{10} , R and g_s are arbitrary and finite; (2) $\phi_4 \rightarrow \phi_0 = 0$ and Λ_{10} is small and R is large.

4 Cosmological Constant Due to $\Lambda_d < 0$

Let the cosmological constant of the bulk space be negative. Therefore, the minimum of the potential (11) takes place at $\phi_{p+1}(\xi^\alpha) = \varphi_{p+1}$, where

$$e^{2\varphi_{p+1}} = -\frac{\Lambda_d}{4(p+1)\pi G_{p+1} T_p}. \tag{15}$$

The minimum of the potential is given by

$$\Lambda_{p+1}^{(d)}(\varphi_{p+1}) = 4\pi(1-p)G_{p+1}T_p \exp\left(\frac{2(p+1)}{p-1}\varphi_{p+1}\right). \tag{16}$$

For $p \geq 2$ this always is nonzero and negative. The ratio of the bulk and brane contributions to (16) is

$$\left| \frac{\Lambda_{\text{bulk}}^{(d)}}{\bar{\Lambda}_{p+1}^{(d)}} \right|_{\phi_{p+1}=\varphi_{p+1}} = \frac{1}{2}(p+1). \tag{17}$$

This shows that for $p \geq 2$ the bulk contribution is greater than the brane contribution.

The potential (11) at $\bar{\phi}_{p+1}$ vanishes, where

$$\bar{\phi}_{p+1} = \varphi_{p+1} + \frac{1}{2} \ln\left(\frac{p+1}{2}\right). \tag{18}$$

This leads to the following form for the potential

$$\Lambda_{p+1}^{(d)}(\phi_{p+1}) = \frac{8\pi G_d T_p}{(2\pi R)^{d-p-1}} \exp\left(\frac{4}{p-1}\phi_{p+1}\right) (e^{2\phi_{p+1}} - e^{2\bar{\phi}_{p+1}}). \tag{19}$$

Therefore, negativity of Λ_d leads to a continues form, from negative to positive values for the cosmological constant of the $(p + 1)$ -dimensional world. In fact, the dynamics of ϕ_{p+1} rolls it toward φ_{p+1} and the parameters in the pre-factor of (19) adjust the rapidity of this rolling.

According to (4) the potential (19) depends on $\sqrt{G_d}$. However, the large values of ϕ_{p+1} generally control small G_d and large radius of compactification R . Thus, the mechanism of vanishing of $\Lambda_{p+1}^{(d)}(\phi_{p+1})$ through the variables G_d and R generally can turn off by ϕ_{p+1} .

4.1 Four-Dimensional Universe

Look at the D3-brane of the type IIB superstring theory in the regime $\Lambda_{10} < 0$. According to (19) the cosmological potential of the 4-dimensional world can be written as

$$\Lambda_4^{(10)}(\phi_4) = \frac{g_s \alpha'^2}{8\pi^2 R^6} e^{2\phi_4} (e^{2\phi_4} - e^{2\bar{\phi}_4}). \tag{20}$$

Since the observed value of the cosmological constant of the 4-dimensional world is positive, the scalar field ϕ_4 of the today’s universe is away from $\bar{\phi}_4$. In other words, we have $\phi_4|_{\text{present}} > \bar{\phi}_4$. After this, we receive $\phi_4 = \bar{\phi}_4$ with $\Lambda_4^{(10)} = 0$. Then, ϕ_4 will go toward φ_4 with negative cosmological constant. Finally, the universe will obtain the following minimum

$$\Lambda_4^{(10)}(\varphi_4) = -\frac{g_s \alpha'^2}{8\pi^2 R^6} e^{4\varphi_4}. \tag{21}$$

Since φ_4 is the stable point, the final cosmological constant of the universe will be negative.

As discussed by Weinberg [23], a large negative cosmological constant forces the universe to collapse rapidly. In the same way, a large positive cosmological constant causes all matters in the universe to disperse. These inspire that the factor $\frac{g_s \alpha'^2}{8\pi^2 R^6}$ should sufficiently be small to dominate on the ϕ_4 -part of (20).

5 The Cosmological Constant of the Dual World

Introducing (4) and (10) into (11) leads to

$$\Lambda_{p+1}^{(d)} = \Lambda_d \exp\left(\frac{4}{p-1}\phi_{p+1}\right) + \frac{8^{1-d/4}}{\pi^{d/2}} \sqrt{G_d} \alpha'^{(d-2p-4)/4} R^{p+1-d} \exp\left(\frac{2(p+1)}{p-1}\phi_{p+1}\right). \tag{22}$$

This form of the cosmological constant explicitly reveals the adjustable parameters Λ_d , G_d , d , p and R and also dynamical variable ϕ_{p+1} .

Now let Dp' -brane, with $p' = d - p - 4$, be dual of the Dp -brane. Assume that the Dp' -brane also is non-compact. This implies that the dual brane lives in another d -dimensional bulk space with $d - p' - 1 = p + 3$ compact directions. According to the number of the compact directions, the bulk spaces should be different. The cosmological constant of the dual world, for both cases $\Lambda_d > 0$ and $\Lambda_d < 0$, is given by (22) with $p \rightarrow p' = d - p - 4$,

$$\Lambda_{p'+1}^{(d)} = \Lambda_d \exp\left(\frac{4}{d - p - 5} \phi_{p'+1}\right) + \frac{8^{1-d/4}}{\pi^{d/2}} \sqrt{G_d} \frac{1}{\alpha'^{(d-2p-4)/4}} \frac{1}{R^{p+3}} \exp\left(\frac{2(d - p - 3)}{d - p - 5} \phi_{p'+1}\right). \tag{23}$$

For the self-dual brane, i.e. $p = p' = d/2 - 2$, both systems are the same. Therefore, the cosmological constants (22) and (23) reduce to

$$\Lambda_{p+1}^{(d)} = \Lambda_d \exp\left(\frac{8}{d - 6} \phi_{p+1}\right) + \frac{8^{1-d/4}}{\pi^{d/2}} \sqrt{G_d} \frac{1}{R^{d/2+1}} \exp\left(\frac{2(d - 2)}{d - 6} \phi_{p+1}\right). \tag{24}$$

This depends on the string coupling g_s and string length scale α' through G_d . For example, for $d = 10$ we have $G_{10} = 8\pi^6 g_s^2 \alpha'^4$.

5.1 Dirac Quantization of the Cosmological Constants for the Case $\Lambda_d < 0$

For the R - R charges μ_p and $\mu_{p'}$ corresponding to the Dp -brane and its dual brane, there is the Dirac quantization

$$\mu_p \mu_{p'} = \frac{n\pi}{\kappa_d^2}, \tag{25}$$

for some integer n . The charge μ_p is related to the Dp -brane tension by $\mu_p = g_s T_p$. Thus, according to (19) the product of $\Lambda_{p+1}^{(d)}$ and $\Lambda_{p'+1}^{(d)}$ is a quantized quantity

$$\Lambda_{p+1}^{(d)} \Lambda_{p'+1}^{(d)} = n \frac{8\pi^2 G_d}{g_s^2 (2\pi R)^{d+2}} \exp\left(\frac{4}{p - 1} \phi_{p+1} + \frac{4}{p' - 1} \phi_{p'+1}\right) \times (e^{2\phi_{p+1}} - e^{2\bar{\phi}_{p+1}})(e^{2\phi_{p'+1}} - e^{2\bar{\phi}_{p'+1}}). \tag{26}$$

In the language of energy density, this expression can be interpreted as the quantization of vacuum energy density of each world in terms of the unit vacuum energy density of its dual world.

6 Conclusions and Summary

We derived the effective cosmological constant for a $(p + 1)$ -dimensional world which lives in the d -dimensional partially compact bulk space. We observed that finiteness of the vacuum energy density also imposes the compactness of the extra dimensions. The effects of the compactification on the bulk and brane components of the cosmological constant are different. For the large extra dimensions the brane-component goes to zero.

Since we used the Einstein's frame, a scalar field extracted from the compactification, appeared in the cosmological constant. By adjusting the parameters of the model, the cosmological constant through the dynamics of this scalar field, goes to the demanded value. For example, the large values of the above scalar field dominate and hence a nonzero cosmological constant will be obtained.

For the positive bulk cosmological constant Λ_d , the cosmological constant of the $(p+1)$ -dimensional world is always positive. In this case, when the scalar field is in $-\infty$, we obtain $\Lambda_{p+1}^{(d)} \rightarrow 0^+$. For the negative Λ_d , the cosmological constant continuously changes from the positive values to the negative values with a negative minimum as a stable point. Therefore, in some points it is positive and very small. In a special point it also vanishes.

We observe that for $\Lambda_d < 0$ the cosmological constants of the universe and the dual universe obey the Dirac quantization. In fact, this kind of quantization is incorporated with the energy quantization.

As an interesting example, for the D3-brane of the type IIB theory, the dynamics of ϕ_4 enables us to obtain a tiny cosmological constant of our world for both cases $\Lambda_{10} > 0$ and $\Lambda_{10} < 0$. Adjusting the parameters helps us to modify the rapidity of the rolling scalar ϕ_4 . However, there exists a solution $\Lambda_4^{(10)} = 0$ only for the negative Λ_{10} . When the factor $\frac{g_s \alpha'^2}{8\pi^2 R^6}$ is very small, rapid collapse and rapid dispersion do not take place in the universe.

References

- Weinberg, S.: The cosmological constant problem. *Rev. Mod. Phys.* **61**, 1 (1989)
- Witten, E.: The cosmological constant from the viewpoint of string theory. hep-ph/0002297
- Arkani-Hamed, N., Dimopoulos, S., Kaloper, N., Sundrum, R.: A small cosmological constant from a large extra dimension. *Phys. Lett. B* **480**, 193 (2000). hep-th/0001197
- Arkani-Hamed, N., Dimopoulos, S., Dvali, G.: The hierarchy problem and new dimensions at a millimeter. *Phys. Lett. B* **429**, 263 (1998). hep-ph/9803315
- Kachru, S., Schulz, M.B., Silverstein, E.: Self-tuning flat domain walls in 5d gravity and string theory. *Phys. Rev. D* **62**, 045021 (2000). hep-th/0001206
- Randall, L., Sundrum, R.: A large mass hierarchy from a small extra dimension. *Phys. Rev. Lett.* **83**, 3370 (1999). hep-ph/9905221
- Randall, L., Sundrum, R.: An alternative to compactification. *Phys. Rev. Lett.* **83**, 4690 (1999). hep-th/9906064
- Antoniadis, I., Arkani-Hamed, N., Dimopoulos, S., Dvali, G.: New dimensions at a millimeter to a Fermi and superstrings at a TeV. *Phys. Lett. B* **436**, 257 (1998). hep-ph/9804398
- de Alwis, S.P., Flournoy, A.T., Irges, N.: Brane worlds, the cosmological constant and string theory. *JHEP* **0101**, 027 (2001). hep-th/0004125
- Burgess, C.P., Myers, R.C., Quevedo, F.: A naturally small cosmological constant on the brane? *Phys. Lett. B* **495**, 384 (2000). hep-th/9911164
- Forste, S., Lalak, Z., Lavignac, S., Nilles, H.P.: The cosmological constant problem from a brane-world perspective. *JHEP* **0009**, 034 (2000). hep-th/0006139
- Carroll, S.M.: The cosmological constant. *Living Rev. Rel.* **4**, 1 (2001). astro-ph/0004075
- Padmanabhan, T.: Cosmological constant: The weight of the vacuum. *Phys. Rep.* **380**, 235 (2003). hep-th/0212290
- Nobbenhuis, S.: Categorizing different approaches to the cosmological constant problem. gr-qc/0411093
- Polchinski, J.: The cosmological constant and the string landscape. hep-th/0603249
- Krause, A.: A small cosmological constant and backreaction of non-finetuned parameters. *JHEP* **0309**, 016 (2003). hep-th/0007233
- Easther, R., Maeda, K., Wands, D.: Tree-level string cosmology. *Phys. Rev. D* **53**, 4247 (1996). hep-th/9509074
- Banks, T.: T C P, quantum gravity, the cosmological constant and all that. . . *Nucl. Phys. B* **249**, 332 (1985)
- Linde, A.D.: Inflation and quantum cosmology. In: Hawking, S.W., Israel, W. (eds.) *Three Hundred Years of Gravitation*. Cambridge University Press, Cambridge (1987)

20. Gomez, C., Janssen, B., Silva, P.: Dilatonic Randall-Sundrum theory and renormalisation group. JHEP **0004**, 024 (2000). hep-th/0002042
21. Torii, T., Shiromizu, T.: Cosmological constant, dilaton field and Freund-Rubin compactification. Phys. Lett. B **551**, 161 (2003). hep-th/0210002
22. Ito, M.: Scalar potential from de Sitter brane in 5D and effective cosmological constant. JHEP **0406**, 023 (2004). hep-th/0311006
23. Weinberg, S.: Anthropic bound on the cosmological constant. Phys. Rev. Lett. **59**, 2607 (1987)